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Stress Analysis of Rectangular Boxes Using Fem

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ABSTRACT
Extensive experimental & theoretical contributions have been made to the study of open box structures, but few references dealing with closed boxes have been found. When a rectangular box structure is subjected to certain pressure, stress analysis of rectangular box is necessary to avoid the failure during working condition. In this work, it is proposed to evaluate the stresses in rectangular box by changing L/B ratios 1, 1.5, 2 for different thicknes of 2.5, 5, 7.5 mm & varying fillet radius, using finite element method.To validate finite element stresses, it is necessary to compare these stresses with analytical approach. From the FE analysis of rectangular box, it is seen that cubical box having the lesser stresses & better for stress distribution due to symmetry.The stiffners further reduces the stresses in boxes.

I. INTRODUCTION

The knowledge of stresses & strains in box-shaped structures subjected to different types of loads are of considerable interest to engineers. The important application can be found in the use of this kind of structure in under water engineering & pressure vessel.

Because of the complicated deformation, the research for a rectangular box relies mainly on finite element method & experiments. A pressure vessel is closed container designed to hold gases or liquids at a pressure substantially different from the gauge pressure. The pressure vessels are designed with great care because rupture of pressure vessels means an explosion which may cause loss of life & property. Any pressure vessel in-service poses extreme potential danger due to the high pressure & varying operating temperature, hence there should be no complacency about the risks.

Comparisons of the rectangular vessels with the equivalent size cylindrical vessels indicate that the former are rather inefficient. Cylindrical vessels will sustain considerably higher pressures, for the same wall thickness & size. However, practical consideration will often force the designer to select a rectangular shape as shown in Fig.1 as the best available option.

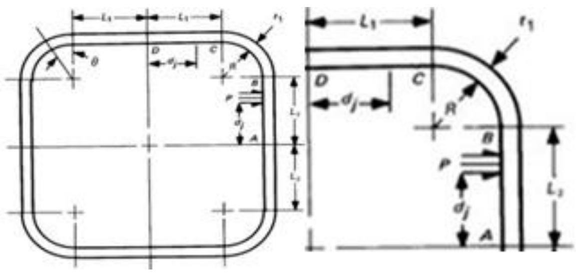

Fig.1: Cross section of rectangular box having uniform thickness & varying fillet radius

The present analysis uses two different approaches, finite element methodology & analytical method. Analytical stress calculations are carried out using ASME section 8, Appendix 13 [6]. For the analysis of rectangualr box at different location i.e., D, A, C, B & at Corner for rectangular box.

Due to symmetry about axis A-A & C-C it will be convenient to analyze one quadrant & this quadrant is in equilibrium under the action of loads & moments as shown in Fig.1. Membrane & bending stress are evaluated to determine the value of minimum stresses occurred at these particular locations & analysing its behaviour under the different cases.

For the Analysis of Rectangular Boxes following cases are considered,

Case 1: Length 100 mm & Breadth 100 mm i.e., L/B =1 for thickness 2.5, 5, 7.5 mm respectively with varying fillet radius.

Case 2: Length 150 mm & Breadth 100 mm i.e., L/B = 1.5 for thickness 2.5, 5, 7.5 respectively with varying fillet radius.

Case 3: Length 200 mm & Breadth 100 mm i.e., L/B = 2 for thickness 2.5, 5, 7.5 mm respectively with varying fillet radius.

Equations are considered from ASME section viii, Appendix 13, which is used to determine minimum wall thickness & design pressure. ASME section viii is the construction code for the pressure vessel. [7]

Total stresses are Maximum at the surfaces where tensile stresses due to the bending moment occur

Modulus of Elasticity, Poissons Ratio & Internal Pressure are 200 x 10^3 Mpa, ν= 0.3, P=1 Mpa respectively.

The Analytical stress calculations for rectangular box are performed using following relations.

1] MEMBRANE STRESS

Short - side plates

$(S_m)_C = (S_m)_D = P\,(R+L_2) / t_1$

Long side plat

$(S_m)_A = (S_m)_B = P\,(L_1+R) / t_1$

Corner sections

$(S_m)_{B-C} = P/t_1 \left(\sqrt{L_2^2 + L_1^2} + R\right)$

2] BENDING STRESS

Short side plates

$(S_b)_C = \pm\, c/2I_1 \times [2M_A + P\,(2RL_2 - 2RL_1 + L_2^2)]$

$(S_b)_D = \pm\, c/2I_1\, [2M_A + P\,(L_2^2 + 2RL_2 - 2RL_1 - L_1^2)]$

Long side plates

$(S_b)_A = M_A c / I_1$

$(S_b)_B = \pm c/2I_1\, (2M_A + PL_2^2)$

Corner sections

$(S_b)_{B-C} = M_r C / I_1$

Total stress = Membrane stress + Bending stress

II. MODELLING & STRESS ANALYSIS OF RECTANGULAR BOXES BY FE APPROACH

The stresses in rectangular box under internal pressure for different thickness & varying fillet radius & stresses induced at various locations are evaluated & presented as follows.

2.1 STRESS ANALYSIS OF RECTANGULAR BOX FOR L/B =1

i.e., (For Length -100 mm & Breadth -100 mm)

For ratio 1, it is seen that, for fillet radius 0 to 47.5 mm max Von-mises stresses found only at corner.

2.1.1 CASE 1: RECTANGULAR BOX OF 2.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1

In this case, 2.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.2 & Stresses are shown in table 1, graph shown in Fig.3.

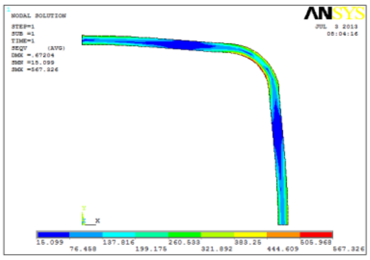

Fig.2: Von-mises stress contour of box 2.5 mm thickness & 10 mm fillet radius

Table 1: Max Von-mises stresses in rectangular box having 2.5 mm thickness at Corner with varying fillet radius for L/B=1

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	784	748.870	4.59
2	883.88	702.3	20.54
4	749.63	657.06	12.34
6	673.61	613.144	8.976
10	567.33	529.368	6.691
12	520.65	489.515	5.979
18	396.53	378.167	4.630
24	291.47	279.401	4.140
28	229.55	220.616	3.891
35	136.95	131.755	3.792
40	82.648	79.4695	3.845
47.5	20.021	19	5.099

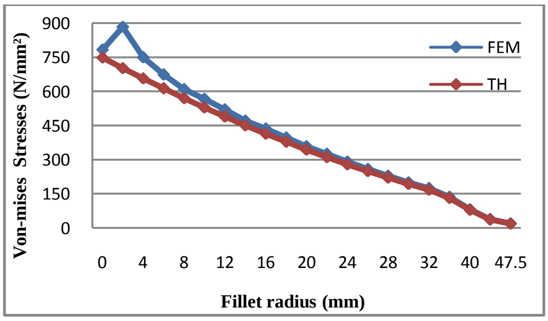

Fig.3: Comparative graphical results for FEM & ANALYTICAL at corner

2.1.2 CASE 2: RECTANGULAR BOX OF 5 MM THICKNESS & VARYING RADIUS FOR L/B =1

In this case, 5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.4 & Stresses are shown in Table 2, graph shown in Fig.5.

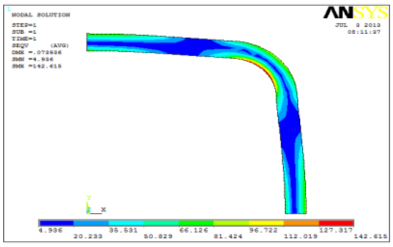

Fig.4: Von-mises stress contour of box 5 mm thickness & 10 mm fillet radius

Table 2: Max Von-mises stresses in rectangular box having 5 mm thickness at Corner with varying fillet radius for L/B=1

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	208.79	174.73	16.66
2	233.79	163.62	30.01
4	202.78	152.85	24.62
6	176.38	142.40	19.26
10	142.61	122.52	14.08
12	128.96	113.10	12.29
18	96.434	86.873	9.91
24	70.055	63.803	8.92
28	54.817	50.205	8.41
35	32.639	29.927	8.30
40	20.01	18.252	8.78
45	10.053	9	10.47

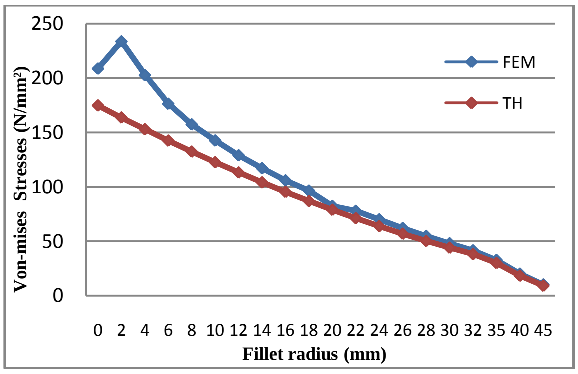

Fig.5: Comparative graphical results for FEM & ANALYTICAL at corner

2.1.3 CASE 3: RECTANGULAR BOX OF 7.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1

In this case, 7.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.6 & Stresses are shown in Table 3, graph shown in Fig.7

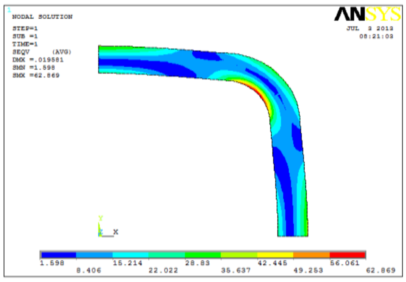

Fig.6: Von-mises stress contour of box 7.5 mm thickness &10 mm fillet radius

Table 3: Max Von-mises stresses in rectangular box having 7.5 mm thickness at Corner with varying fillet radius for L/B=1

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	95.203	72.233	24.15
2	122.91	67.51	45.06
4	95.117	62.964	33.80
6	80.578	58.561	27.32
10	62.661	50.212	19.86
12	56.678	46.249	18.39
18	41.646	35.316	15.19
24	29.921	25.784	13.82
28	23.408	20.249	13.49
35	14.034	12.082	13.90
40	8.853	7.522	15.02
42.5	6.74	5.66	16.02

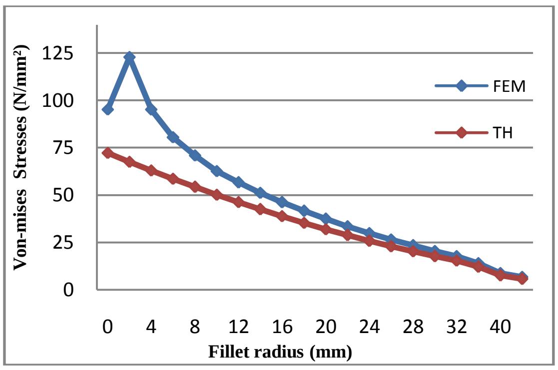

Fig.7: Comparative graphical results for FEM & ANALYTICAL at corner

2.2 STRESS ANALYSIS OF RECTANGULAR BOX FOR L/B =1.5

i.e., (For Length -150 mm & Breadth -100 mm)

For ratio 1.5 it is seen that, for fillet radius 0 to 4 mm, Max Von-mises stresses found at corner but as fillet radius goes on increasing, Max Von-mises stresses shift to location A, it may be due to maximum bending moment acting along the long side of rectangle.

2.2.1 CASE 1: RECTANGULAR BOX OF 2.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1.5

In this case, 2.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.8 & Stresses are shown in Table 4, graph shown in Fig.9.

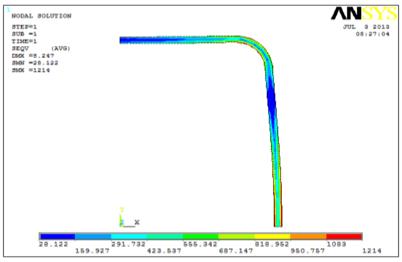

Fig.8: Von-mises stress contour of box 2.5 mm thickness & 10 mm fillet radius

Table 4: Max Von-mises stresses in rectangular box having 2.5 mm thickness at location A with varying fillet radius for L/B=1.5

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	1278.6	1240	3.01
2	1257	1230.34	2.12
4	1249.7	1220.34	2.37
6	1236.9	1208.9	2.25
10	1209.2	1184.84	2.01
12	1195.5	1171.71	1.98
18	1150.1	1128.01	1.92
24	1094	1077	1.49
28	1061.8	1040.34	2.02
35	987.75	967.47	2.05
40	928.14	909.38	2.02

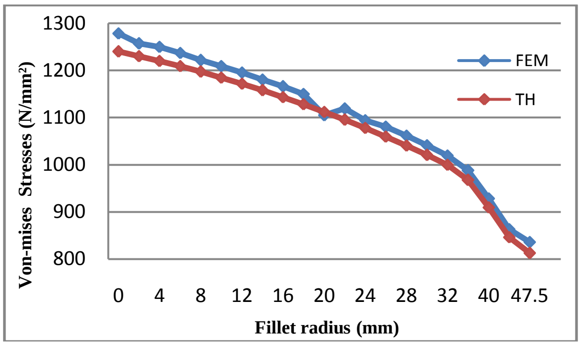

Fig.9: Comparative graphical results for FEM & ANALYTICAL at location A

2.2.2 CASE 2: RECTANGULAR BOX OF 5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B=1.5

In this case, 5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.10 & Stresses are shown in Table 5, graph shown in Fig.11

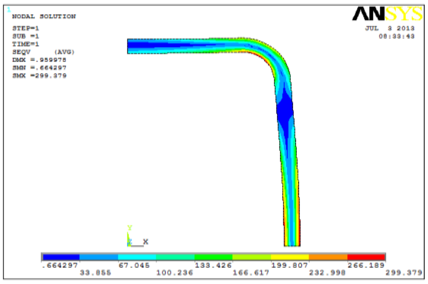

Fig.10: Von-mises stress contour of box 5 mm thickness & 10 mm fillet radius

Table 5: Max Von-mises stresses in rectangular box having 5 mm thickness at location A with varying fillet radius for L/B = 1.5

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	310.83	295	4.83
2	308.3	292.6	5.07
4	305.64	290.15	5.06
6	302.62	287.46	5.00
10	296.02	281.57	4.87
12	292.54	278.36	4.84
18	280.89	267.64	4.71
24	267.59	255.26	4.60
28	257.78	246.05	4.54
35	238.69	228.03	4.46
40	223.51	213.63	4.41

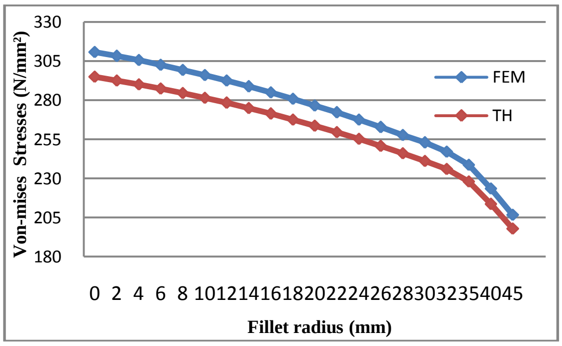

Fig.11: Comparative graphical results for FEM & ANALYTICAL at location A

2.2.3 CASE 3: RECTANGULAR BOX OF 7.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1.5

In this case, 7.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.12 & Stresses are shown in Table 6, graph shown in Fig.13.

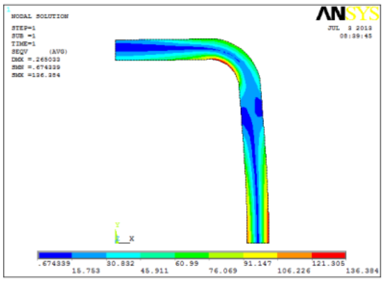

Fig.12: Von-mises stress contour of box 7.5 mm thickness & 10 mm fillet radius

Table 6: Max Von-mises stresses in rectangular box having 7.5 mm thickness at location A with varying fillet radius for L/B = 1.5

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	134.72	124.44	7.62
2	133.66	123.43	7.64
4	132.31	122.35	7.52
6	130.93	121.193	7.43
10	127.91	118.63	7.24
12	126.31	117.24	7.17
18	121.02	112.56	6.98
24	114.95	107.14	6.78
28	110.52	10310	6.70
35	101.85	95.183	6.54
40	95.051	88.83	6.53

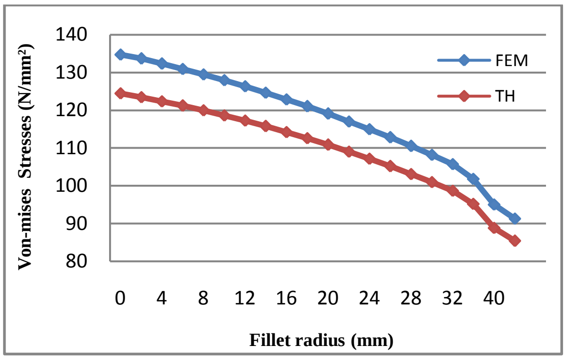

Fig.13: Comparative graphical results for FEM & ANALYTICAL at location A

2.3 STRESS ANALYSIS OF RECTANGULAR BOX FOR L/B =2

i.e. (For Length-200 mm & Breadth -100 mm)

For ratio 2, it is seen that, for fillet radius 0 to 4 mm, Max Von-mises stresses found at corner but as fillet radius goes on increasing, Max Von-mises stresses shift to location A.

2.3.1 CASE 1: RECTANGULAR BOX OF 2.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =2

In this case, 2.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.14 & Stresses are shown in Table 7, graph shown in Fig.15.

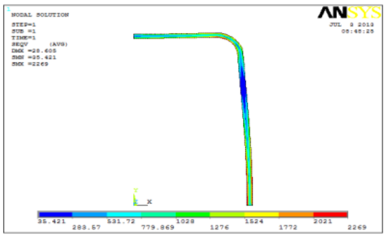

Fig.14: Von-mises stress contour of box 2.5 mm thickness & 10 mm fillet radius

Table 7: Max Von-mises stresses in rectangular box having 2.5 mm thickness at location A with varying fillet radius for L/B = 2

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	2362	2300	2.66
2	2332.3	2286.13	1.97
4	2316.7	2271.56	1.94
6	2299.5	2256.28	1.87
10	2266.1	2223.54	1.87

12	2248	2206.08	1.86
18	2189.2	2149.2	1.82
24	2119.5	2085.59	1.59
28	2071.2	2039.29	1.54
35	1986.9	1950.61	1.82
40	1914.2	1881.16	1.72

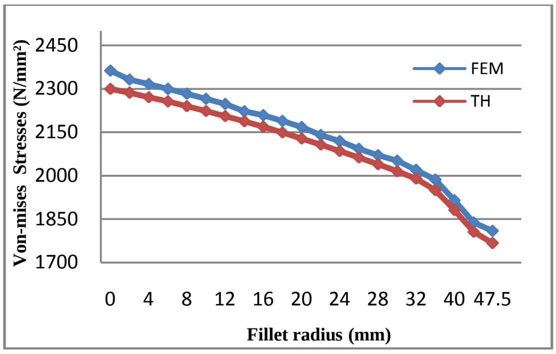

Fig.15: Comparative graphical results for FEM & ANALYTICAL at location A

2.3.2: CASE 2: RECTANGULAR BOX OF 5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =2

In this case, 5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.16 & Stresses are shown in Table 8, graph shown in Fig.17.

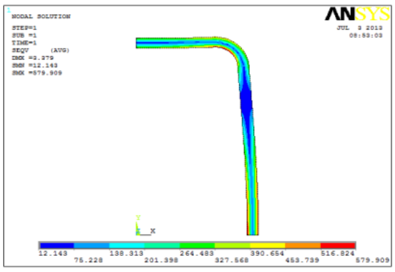

Fig.16: Von-mises stress contour of box 5 mm thickness & 10 mm fillet radius

Table 8: Max Von-mises stresses in rectangular box having 5 mm thickness at location A with varying fillet radius for L/B = 2

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	574.55	550	4.27
2	571	546.58	4.27
4	567.38	542.99	4.29
6	563.11	539.22	4.24
10	554.42	531.14	4.19
12	549.6	526.82	4.14
14	545.05	522.32	4.16
18	534.88	512.75	4.13
24	518.19	496.97	4.09
28	506.15	485.47	4.08
35	483.09	463.41	4.07
40	465.06	446.11	4.07

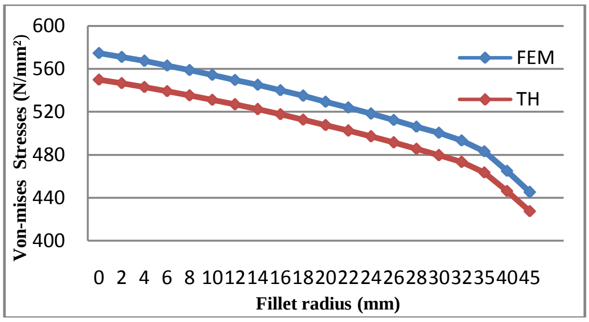

Fig.17: Comparative graphical results for FEM & ANALYTICAL at location A

2.3.3 CASE 3: RECTANGULAR BOX OF 7.5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B = 2

In this case, 7.5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von-mises stress contours are shown in Fig.18 & Stresses are shown in Table 9, graph shown in Fig 19.

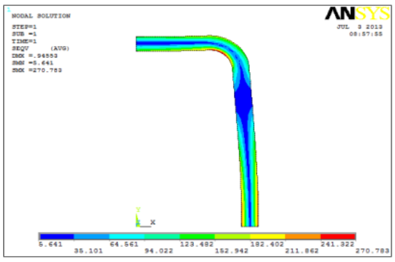

Fig.18: Von-mises stress contour of box 7.5 mm thickness & 10 mm fillet radius

Table 9: Max Von-mises stresses in rectangular box having 7.5 mm thickness at location A with varying fillet radius for L/B = 2

Fillet Radius In mm	FE Approach In Mpa	Analytical Approach In Mpa	% Error
0	249.5	233.333	6.479
2	247.97	231.839	6.504
4	246.07	230.266	6.422
6	244.22	228.613	6.390
10	240.21	225.063	6.305
12	238.09	223.165	6.268
14	235.91	221.184	6.242
18	231.3	216.969	6.195

24	223.68	210.005	6.113
28	218.28	204.924	6.118
35	207.83	195.165	6.093
40	199.76	187.49	6.13

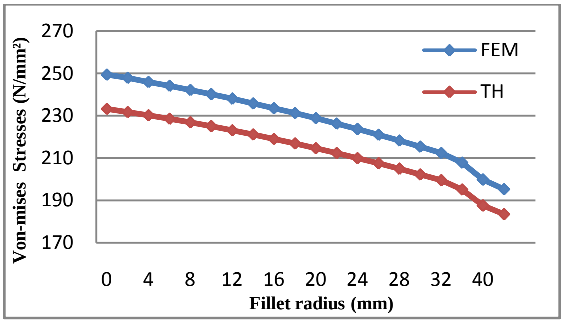

Fig.19: Comparative graphical results for FEM & ANALYTICAL at location A

III. STRESS ANALYSIS OF RECTANGULAR BOX WITH INTERMEDIATE STIFFENER FOR L/B = 1

Stiffeners are the secondary plate or section which are attached to the beam webs or flanges to stiffen them against out of plane deformation. A steel angle or plate attached to a slender beam to prevent its buckling by increasing its stiffness. Stiffness is the rigidity of an object the extent to which resists deformation in response to an applied force.The function of stiffener are for controlling local buckling, connecting bracing or transverse beam & stiffener provides strength to the structure.

In this analysis two different cases are considered for L/B ratio 1 & thickness 5 mm with varying fillet radius & effects of Intermediate & diagonal stiffener in boxes are studied which is shown in Fig.20 & Fig.22 resptively .

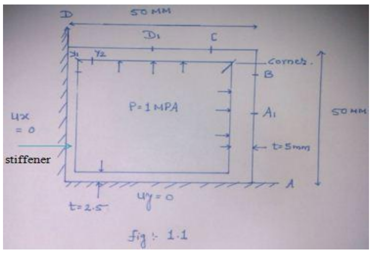

Fig.20: Intermediate Stiffener

3.1 RECTANGULAR BOX WITH INTERMEDIATE STIFFENER OF 5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1

In this case, 5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von Mises stress contours are shown in Fig.21 & Stresses are shown in Table 10.

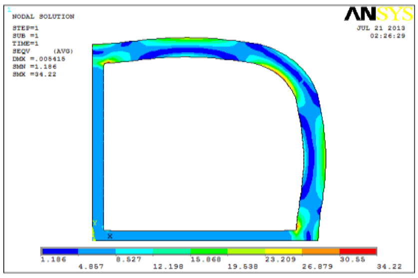

Fig.21: Von-mises stress contour of box 5 mm thickness & 10 mm fillet radius

Table 10: Max Von-mises stresses in rectangular box for Intermediate stiffener having 5 mm thickness at locations D, A, C, B & at Corner with varying fillet radius for L/B =1

Radius in mm	Stresses at D (Mpa)	Stresses at A (Mpa)	Stresses at pt C (Mpa)	Stresses at pt B (Mpa)	Stresses at Corner (Mpa)
0	24.67	24.8	21.2	21.3	48.74
4	22.63	22.7	29.00	30.4	48.02
6	21.39	21.4	21.50	21.0	40.20
10	18.60	18.30	11.12	11.3	30.76
12	17.08	16.8	7.67	8.20	27.81
14	15.49	15.3	7.12	7.04	24.43
18	12.20	12.2	10.1	10.1	19.32
24	9.388	7.13	11.8	11.8	13.20
28	8.546	8.57	11.0	11.0	10.50
35	7.819	7.69	7.12	7.35	6.88
40	8.456	8.45	12.8	12.80	5.190
42.5	9.641	9.63	12.00	13.0	11.40

IV. STRESS ANALYSIS OF RECTANGULAR BOX WITH DIAGONAL STIFFENER FOR L/B= 1

The stresses in rectangular box under internal pressure with diagonal stiffener as shown in Fig.22 are studied for thickness 5 mm & varying fillet radius & stresses induced at various locations are evaluated.

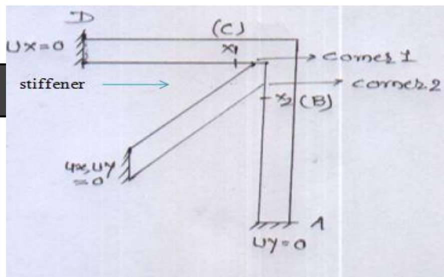

Fig.22: Diagonal Stiffener

4.1 RECTANGULAR BOX WITH DIAGONAL STIFFENER OF 5 MM THICKNESS & VARYING FILLET RADIUS FOR L/B =1

In this case, 5 mm thickness & varying fillet radius is considered. The finite element analysis of rectangular box with fillet radius as per loading & boundary conditions revealed the stress distribution in the form of stress contour. The representative Von Mises stress contours are shown in Fig.23 & Stresses are shown in Table 11.

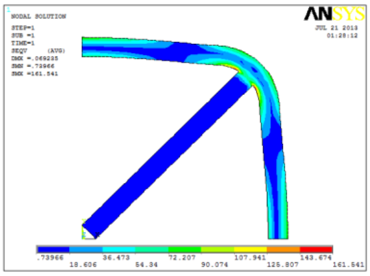

Fig.23: Von-mises stress contour of box 5 mm thickness & 10 mm fillet radius

Table 11: Max Von-mises stresses in rectangular box for Diagonal Stiffener having 5 mm thickness at locations D, A, C, B & at Corner with varying fillet radius for L/B =1

Radius (mm)	Stresses at pt D (Mpa)	Stresses at pt A (Mpa)	Stresses at Corner - 1 (Mpa)	Stresses at Corner -2 (Mpa)	Stresses at pt C (Mpa)	Stresses at pt B (Mpa)
0	82.66	82.60	123.34	124.4	162.92	163.73
4	82.94	82.20	132.14	126.13	161.94	161.55
6	84.84	84.88	129.19	132.19	191.34	192.62
10	82.67	82.59	113.45	113.1	152.26	161.54
12	80.65	80.64	102.1	100.22	141.86	146.98
14	78.31	78.23	89.422	90.21	131.33	133.14
18	71.93	71.80	69.43	68.711	102.34	101.02
24	59.21	59.16	43.721	44.99	65.471	64.38
28	49.89	49.53	33.22	31.374	46.894	46.52
35	33.88	33.88	20.14	20.37	28.596	29.62
40	23.15	23.17	17.02	16.59	23.41	23.82
45	12.65	12.63	14.98	14.67	20.55	20.80

V. CONCLUSION

1 As the L/B ratio is increased i.e., 1, 1.5, 2 for thickness i.e., 2.5, 5, 7.5 mm with varying fillet radius, it is observed that as the L/B ratio increases the maximum Von-mises stresses also increases but if the L/B ratio is kept constant with increasing thickness i.e, 2.5, 5, 7.5 mm & varying fillet radius the Max Von-mises stresses reduces.

2 For L/B ratio 1 & thickness 2.5, 5, 7.5 mm maximum stresses are observed at corner & for ratio 1.5, 2, it is seen that for fillet radius 0 to 4 mm, Max Von-mises stresses found at corner but as fillet radius goes on increasing, Max Von-mises stresses shift to location A, it may be due to maximum bending moment acting along the long side of rectangle.

3 For L/B ratio 1 & thicknesses 2.5, 5, 7.5 mm & fillet radius 47.5, 45, 42.5 mm respectively, it is observed from analytical calculation that, stresses are present only at the Corner but at the

location D, A, C, B bending stress vanishes & only membrane stresses exists.

4 From FE analysis of rectangular box with intermediate & diagonal stiffener, it is observed that, the stresses in the box with stiffener are of lesser magnitude as compared to stresses in box without stiffener.

5 It is seen that cubic box has minimum Von mises stresses as compared to the rectangular box. Thus it is concluded that cubical boxes are better than rectangular boxes w.r.t the stress levels.

6 The Stiffeners are recommended for boxes for lesser magnitudes of stresses in boxes. But it is seen that Intermediate stiffener are better than Diagonal stiffener.

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